Talk for ETH Zürich

Physically Structured Neural Networks
for Smooth and Contact Dynamics

UNIVERSITY OF CAMBRIDGE

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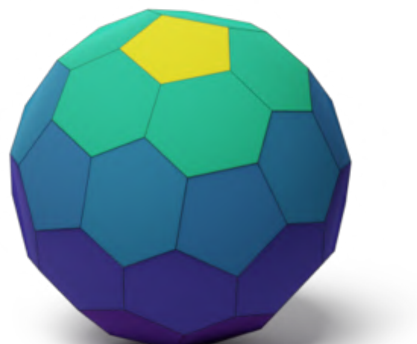
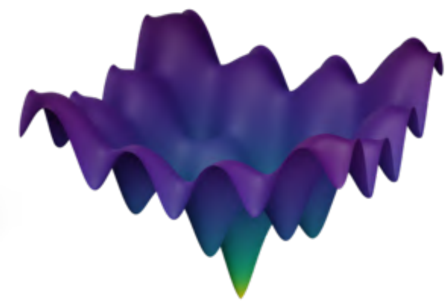
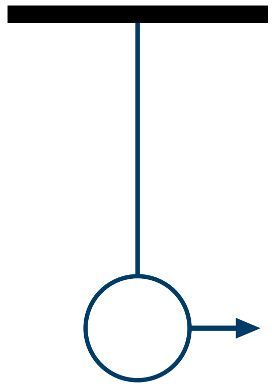
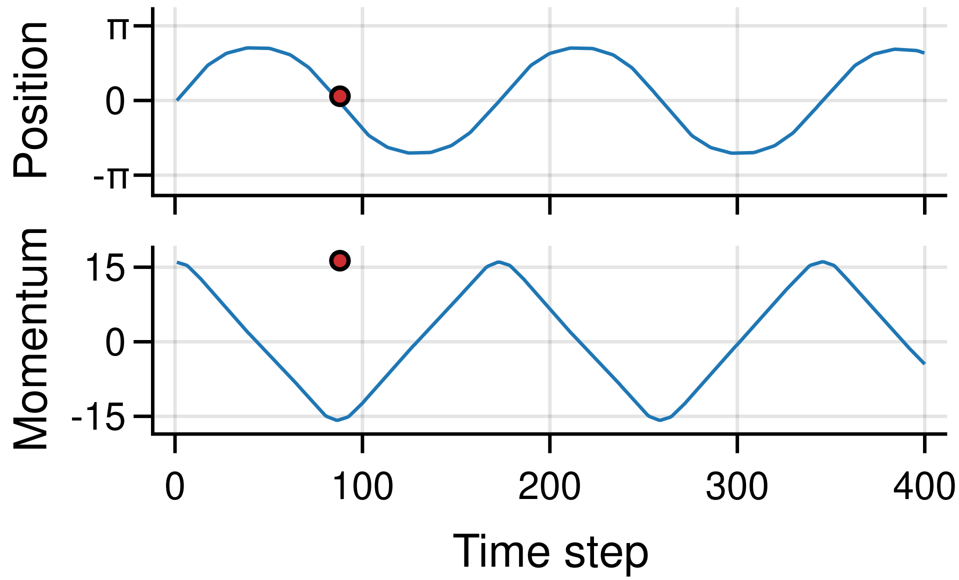
   

This talk: Physically Structured Neural Networks

Most of my work: Gaussian processes and decision-making systems

Physically Structured Neural Networks





x_0 $\quad x_1 = x_0 + f(x_0)\Delta t \quad x_2 = x_1 + f(x_1)\Delta t \quad ..$

Neural Ordinary Differential Equations

$$\underbrace{x_{t+1} = x_t + f(x_t, \theta)}_{\text{residual network}} \quad \rightsquigarrow \quad \underbrace{\frac{\mathrm{d}x_t}{\mathrm{d}t} = f(x_t, \theta)}_{\text{neural ODE}}$$

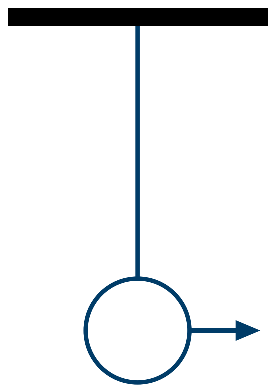
Continuous-time recurrent neural networks:
neural ordinary differential equations

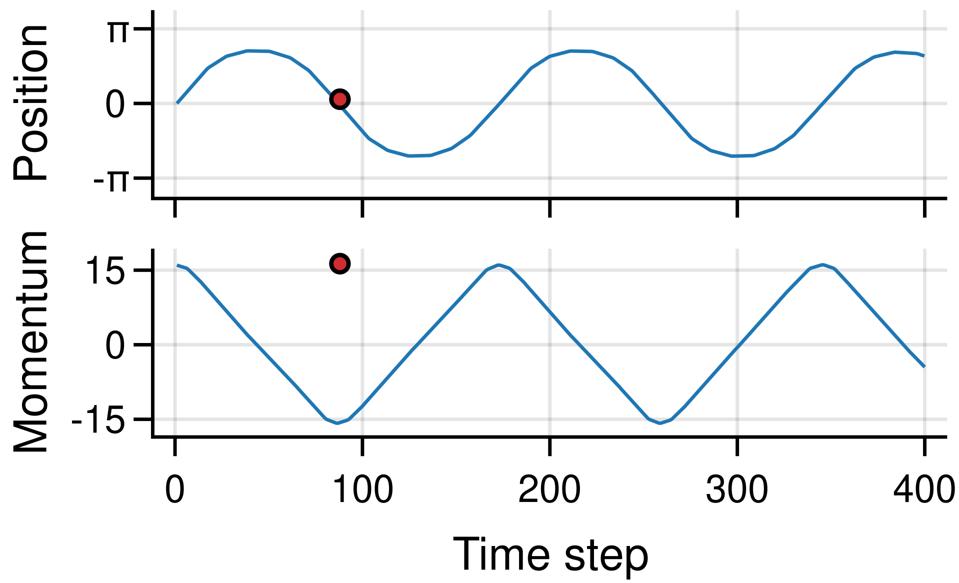
Physically Structured Recurrent Neural Networks

$$\underbrace{\frac{\mathrm{d}x_t}{\mathrm{d}t} = f(x_t, \theta)}_{\text{black-box ODE}} \qquad \rightsquigarrow \qquad \underbrace{\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L_\theta}{\partial \dot{q}} - \frac{\partial L_\theta}{\partial q} = 0}_{\text{Euler-Lagrange equations}}$$

Change the ODE to introduce inductive biases

Physically Structured Recurrent Neural Networks





$$q_0 \qquad q_1 = q_0 + \frac{\partial H}{\partial q}(q_0, p_0)\Delta t \qquad q_2 = q_1 + \frac{\partial H}{\partial q}(q_1, p_1)\Delta t$$

$$p_0 \qquad p_1 = p_0 - \frac{\partial H}{\partial p}(q_1, p_0)\Delta t \qquad p_2 = p_1 - \frac{\partial H}{\partial p}(q_2, p_1)\Delta t \qquad \ldots$$

Variational Integrator Networks
for Physically Structured Embeddings

Steindór Sæmundsson, Alexander Terenin, Katja Hofmann, and Marc Deisenroth




AISTATS 2020

Variational Integrator Networks

$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L_\theta}{\partial \dot{q}} - \frac{\partial L_\theta}{\partial q} = 0}_{\text{Euler-Lagrange equations}} \quad \rightsquigarrow \quad \underbrace{S(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \int_{t_0}^{t_1} L_\theta(\boldsymbol{q}_t, \dot{\boldsymbol{q}}_t)\mathrm{d}t}_{\text{Principle of Least Action}}$$

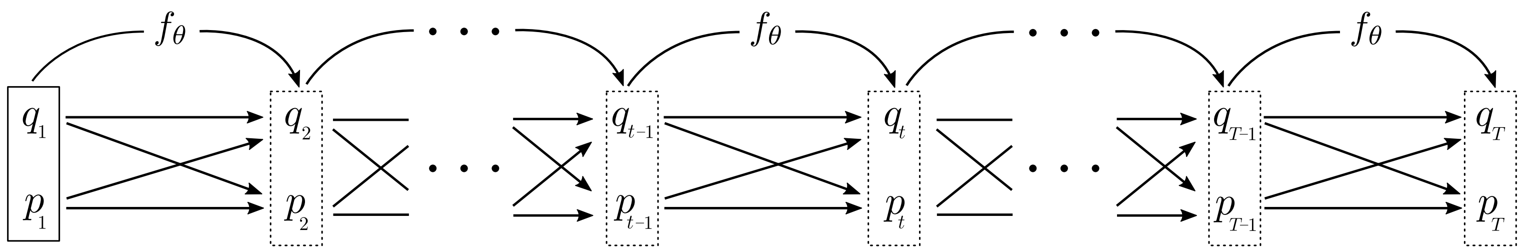
Trajectories of motion are stationary points of action integral

Variational Integrator Networks

$$L_\theta^d(\boldsymbol{q}_t, \boldsymbol{q}_{t+1}, h) \approx \int_t^{t+h} L_\theta(\boldsymbol{q}(\tau), \dot{\boldsymbol{q}}(\tau))\mathrm{d}\tau$$

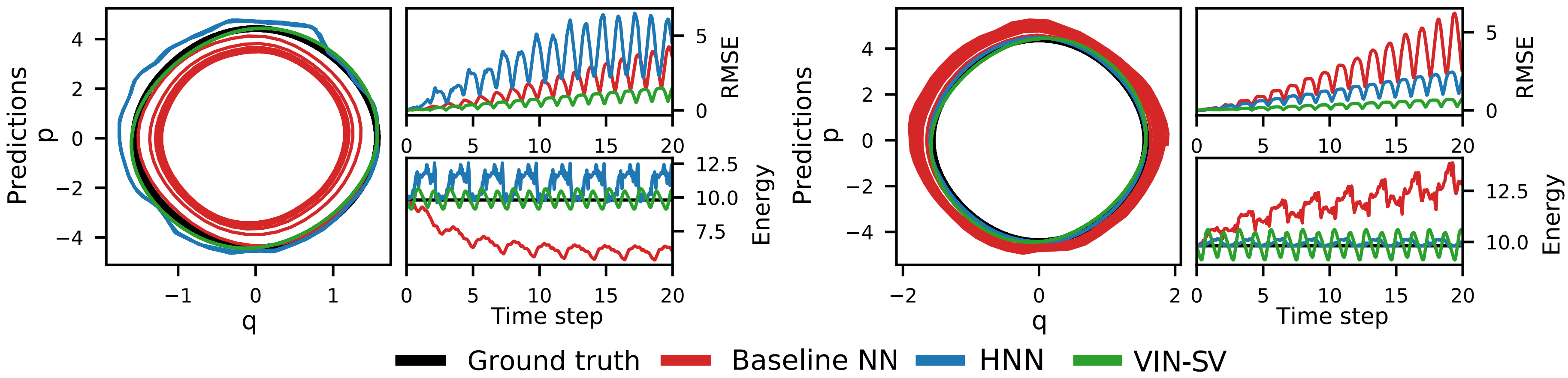
Discretize ODE by discretizing its underlying action integral

Variational Integrator Networks



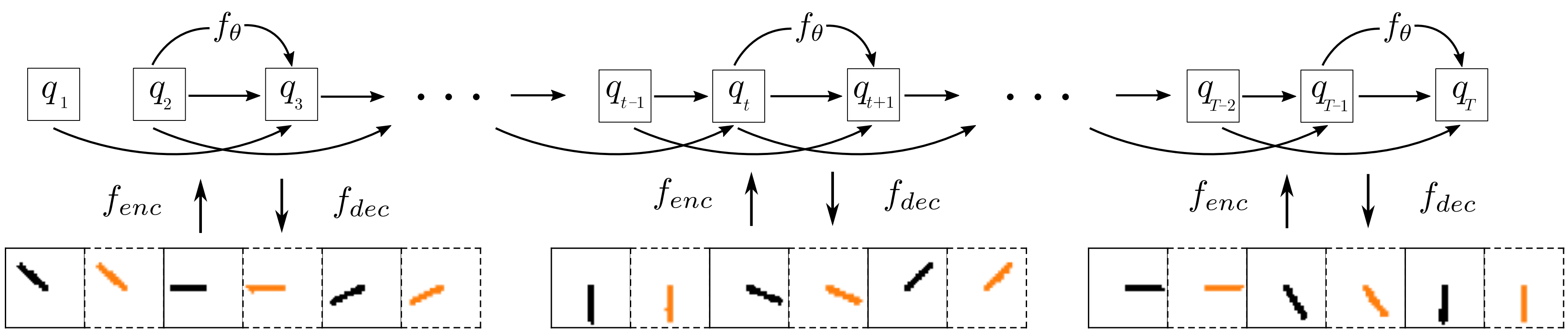
Discrete dynamics lead to neural network architectures

Conservation of Physical Quantities



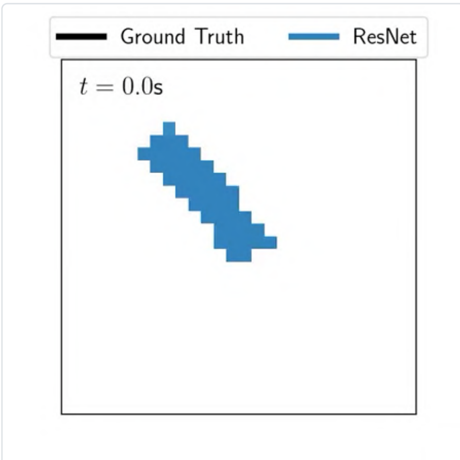
Network learns what quantities to conserve

Physically Structured Autoencoders

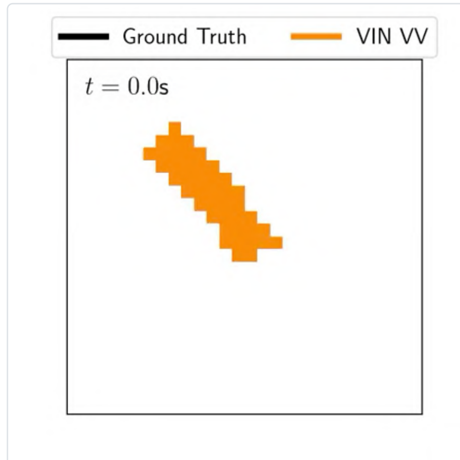


Use inductive biases to efficiently learn from pixels

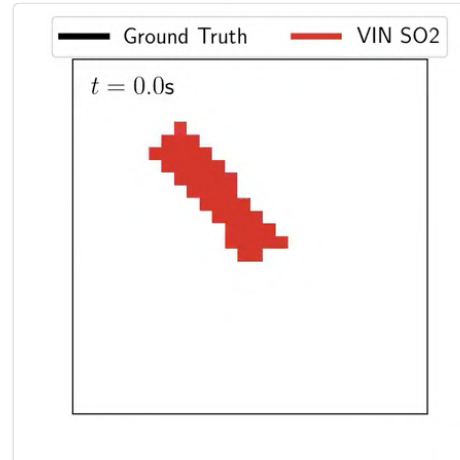
Predicting the motion of a pendulum from pixels



black-box neural network
black: ground truth



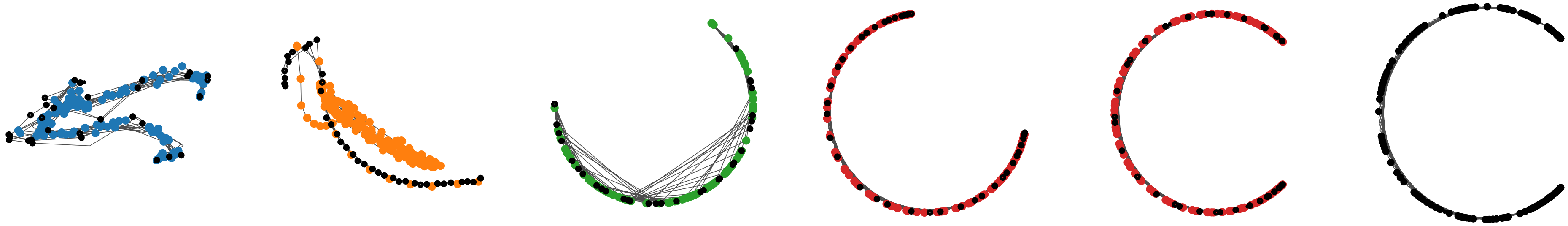
physical structure



physical structure
and correct geometry

Inductive biases drive data-efficiency

Learned Latent Spaces



(a) VAE (b) Dynamic VAE (c) Lie Group VAE (d) VIN-$SO(2)$ (e) VIN-$SO(2)$ (true $\mathbf{M}$) (f) Ground Truth

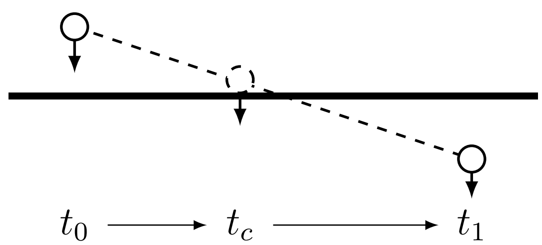
Physical structure yields smoothness in space and time

Learning Contact Dynamics
using Physically Structured Neural Networks

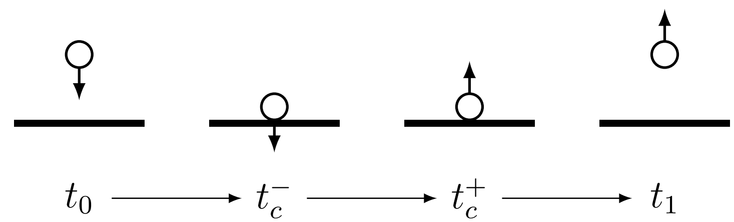
Andreas Hochlehnert, Alexander Terenin, Steindór Sæmundsson, and Marc Deisenroth

AISTATS 2021

Contact Dynamics



(a) Find the contact time t_c



(b) Calculate true trajectory

Collisions lead to discontinuous jumps in velocity

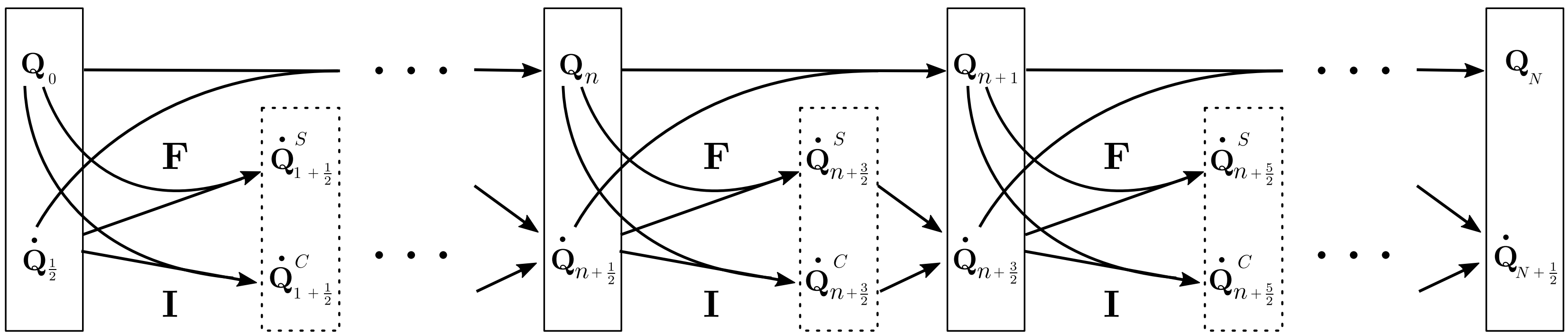
Contact Dynamics

$$\delta S(\boldsymbol{q}, \dot{\boldsymbol{q}}) = 0 \qquad S(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \int_{t_0}^{t_1} L_\theta(\boldsymbol{q}_t, \dot{\boldsymbol{q}}_t)\mathrm{d}t$$

Action integral leads to two components:

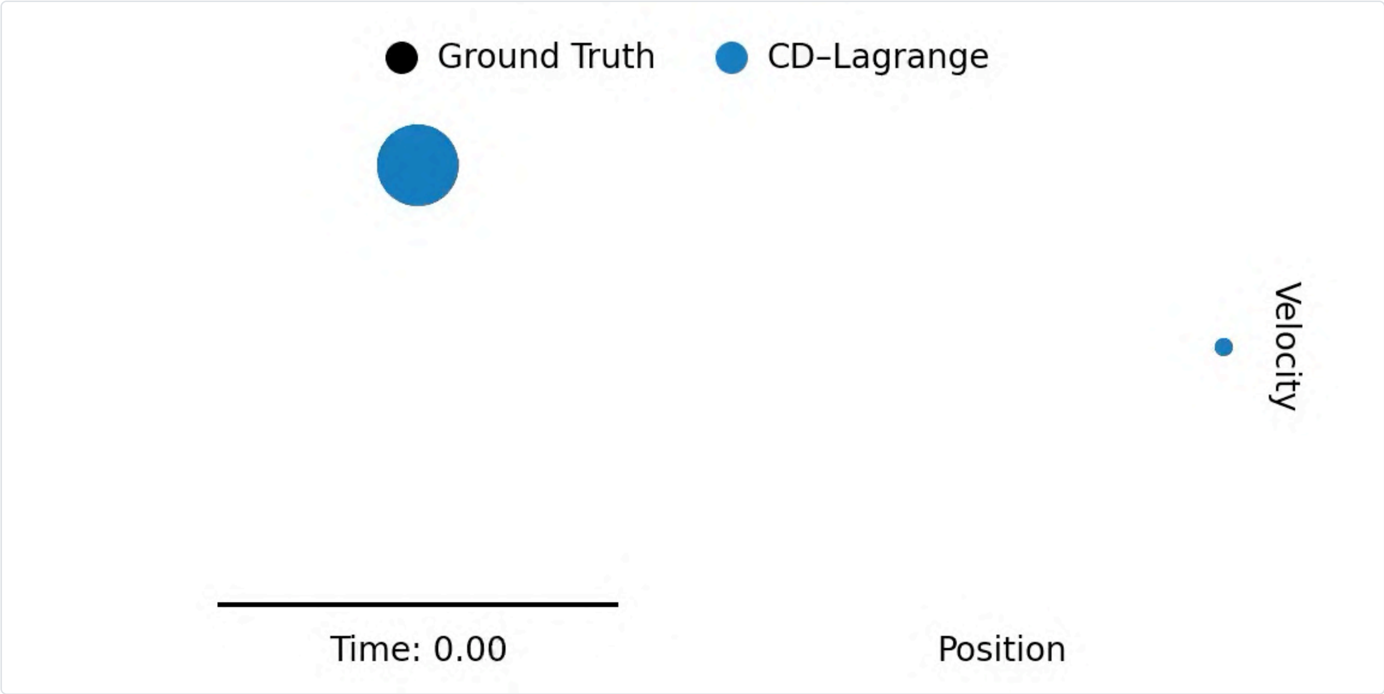
(i) smooth dynamics between contact times
(ii) transfer of momentum at contact events

Contact Dynamics



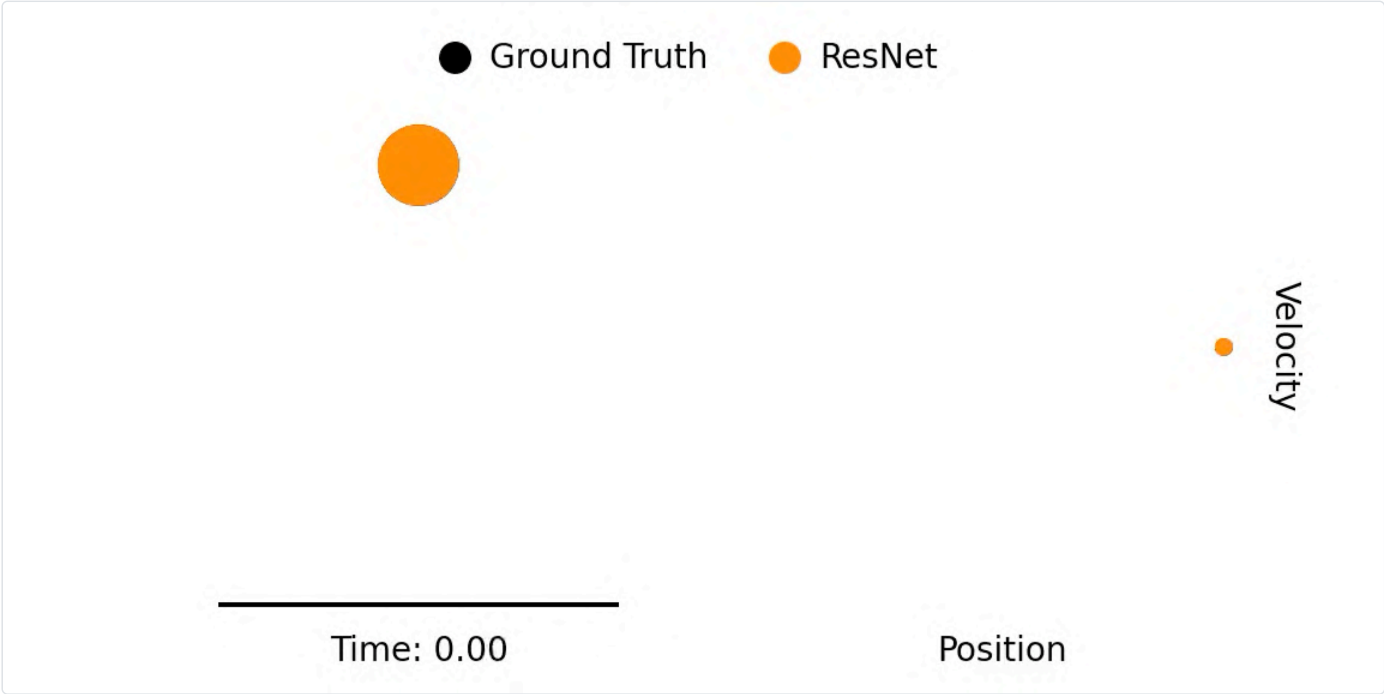
Use neural ODEs to design networks that can model contacts

Contact Dynamics



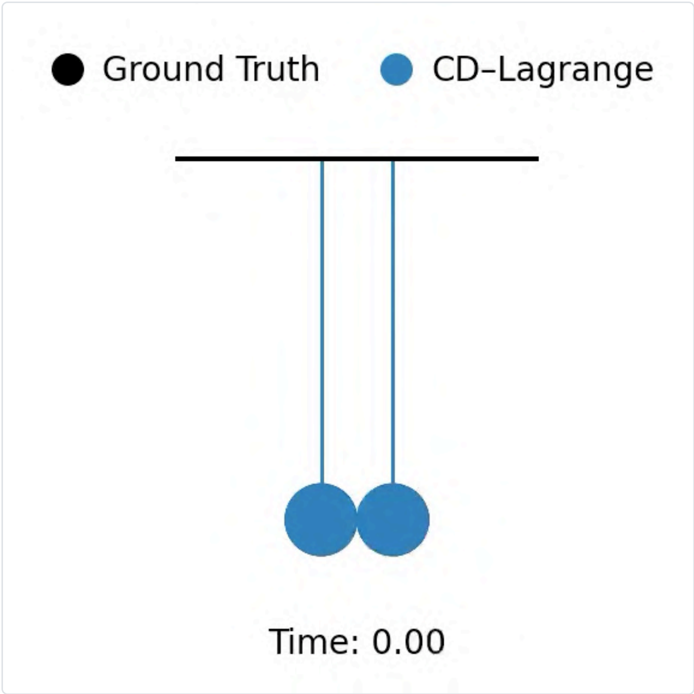
Principled and accurate handling of discontinuity

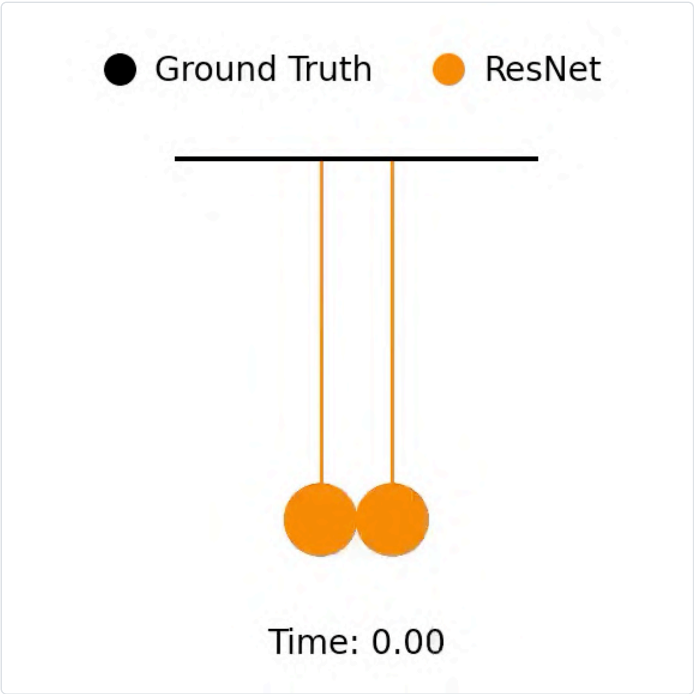
Contact Dynamics



Principled and accurate handling of discontinuity

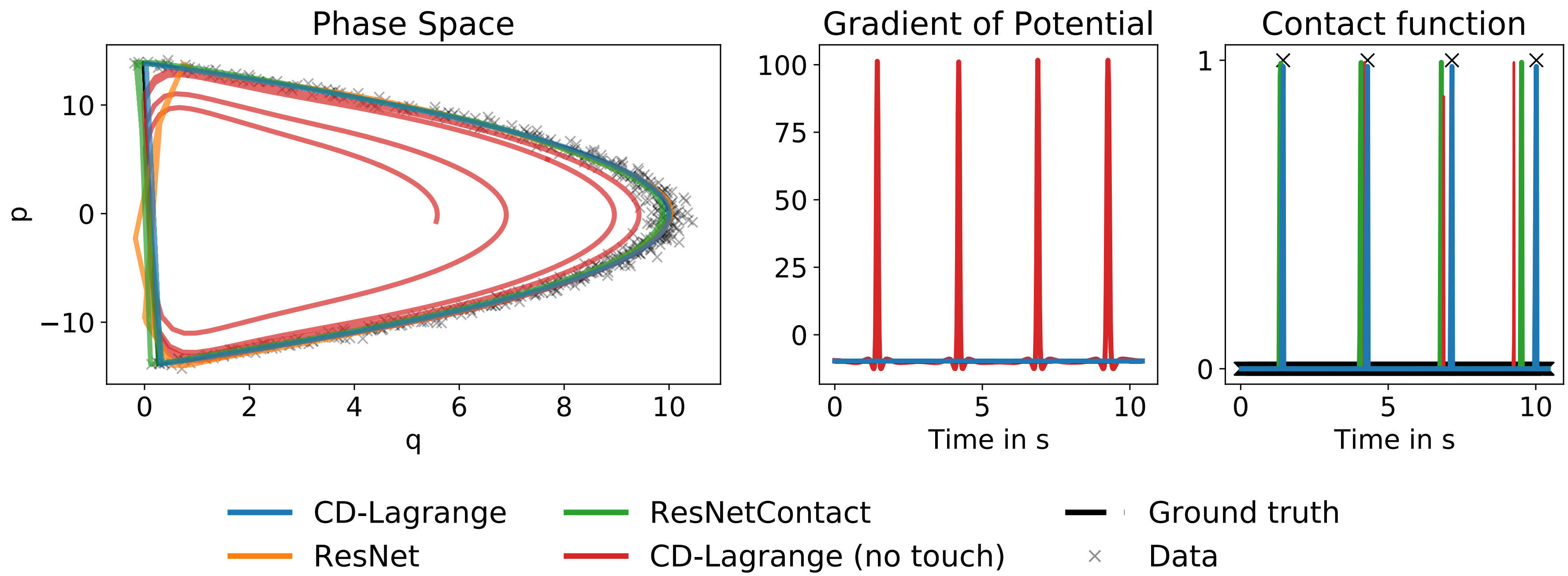
Contact Dynamics





Idealized touch feedback sensor critical for performance

Touch Feedback



Ambiguity: motion can be explained by contact,
or, alternatively, by smooth dynamics

Thank you!

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S. Sæmundsson, A. Terenin, K. Hofmann, M. P. Deisenroth. Variational Integrator Networks for Physically Structured Embeddings. *Artificial Intelligence and Statistics*, 2020.

A. Hochlehnert, A. Terenin, S. Sæmundsson, M. P. Deisenroth. Learning Contact Dynamics using Physically Structured Neural Networks. *Artificial Intelligence and Statistics*, 2021.

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